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ИЗСЛЕДВАНЕ ФЕНОМЕНА НА ПЪЛЗЕНЕТО НА БЕТОНА В КОМБИНИРАНА СТОМАНО-СТОМАНОБЕТОННА ГРЕДА В ПЕРИОД ОТ 100 ГОДИНИ – СРАВНИТЕЛЕН АНАЛИЗ МЕЖДУ ЧИСЛЕНО РЕШЕНИЕ, БАЗИРАНО НА ИНТЕГРАЛНИ УРАВНЕНИЯ НА ВОЛТЕРА И (AAEM) МЕТОД НА БАЖАНТ

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100 YEARS CREEP PHENOMENA IN COMPOSITE STEEL-CONCRETE BEAM –COMPARATIVE ANALYSIS BETWEEN NUMERICAL SOLUTION WITH VOLTERRA INTEGRAL EQUATIONS AND (AAEM) METHOD OF BAŽANT

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Abstract:
The paper presents a precise analysis of the stress-strain behaviour due to creep in statically determinate composite steel-concrete beam according to numerical method in comparison with (AAEM) method of Bažant. The analysis is based on the results obtained by numerical solution with Volterra integral equations and algebraic equation of Bažant, derived for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time "t". The creep law of concrete, according EC2 provisions is used. On the basis of the theory of the viscoelastic body of Arutyunian–Trost-Bažant it is analyzed the migration of stresses from concrete plate to steel beam using two independent Volterra integral equations of the second kind and two independent algebraic equations. The duration of the course of study was adopted 100 years. Developed method will allow soon be confronted methods for calculating the composite beams according to EC4 considering rheology of concrete based on the reduced elastic modulus of concrete.

Keywords:
Steel-Concrete Section, Integral Equations, Rheology, EC2 model, AAEM Method.

1. INTRODUCTION

The time-varying behaviour of composite steel-concrete members under sustained service loads drawn the attention of engineers who were dealing with the problems of their design more than 60 years[3,4,5]. The solution of structural problems involving creep and shrinkage

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phenomena in composite steel-concrete beams has been an important task for engineers since the first formulation of the mathematical model of linear visco-elasticity. From an historical point of view the evolution of the research on this topic, as in many other research fields, has been dramatically influenced by the diffusion of computer technology, starting from the early seventies of past century. Before this event the research was mainly oriented on finding closed form solutions of simple analytic formulations of the creep models. The theory of heritage and the theory of aging, the use of exponential formulations of creep have developed and largely adopted because of their capability to generate closed form solutions of structural problems. Creep and shrinkage have a considerable impact upon the performance of composite beams, causing increased deflection as well as affecting stress distribution. Creep in concrete represents dimensional change in the material under the influence of sustained loading. In general, time-dependent deformation of concrete may severely affect the serviceability, durability and stability of structures[5, 16].

2.ABOUT SOME METHODS FOR TIME- DEPENDENT ANALYSIS OF COMPOSITE STEEL - CONCRETE BEAMS REGARDING RHEOLOGY WITH ACCENTS OF AAEM METOD

2.1. Common considerations

The first works, which give the answer to this problem are based on the Law of Glanville(1933) –Dischinger(1937,1938) - theory of aging[6,7], or also called the rate - of - creep methods, which first formulated a time-dependent stress-strain differential relationship for concrete. Its refinement is known as the improved “Dischinger theory of aging” was proposed from Rüsch-Jungwirth-Hilsdorf(1973)[15] or rate of flow method proposed from England and Illston(1965). Another scientific works which give the answer to this problem used the theory of Maslov(1940)[9] - Arutyunyan(1952)[3] –Prokopovich-(1963)[13]-Aleksandrovskii(1966)[2]. This theory is obviously much more complicated than the „theory of aging”, but the theory of aging, is substantially more complicated than the *effective modulus method(EMM)*. This method connected with the name of McMilan(1916) transform the creep solution for time *t* from the problem of the *”viscoelasticity*” in elastic structural analysis based on the so called effective modlus: $E_{eff} = 1/J(t,\tau) = E(\tau)/1+\varphi(t,\tau)$. For the designers of the composite steel-concrete construction, is better to know that this method is exact only if the loads and stresses in a structure have a single-step history, *which means that they are constant from the moment of first loading*. This fact is more far from true, in internally statically indeterminate system with significant stress redistributions induced by creep or shrinkage, as in the case of composite steel-concrete construction. The same problems we met in the statically indeterminate system, where the changes in the structural system during the construction arise, or if the permanent loads are not applied at once. Many books and papers connected with the Law of Dischinger represent one independent group of normal forces $N_{c,r}(t)$, $N_{a,r}(t)$ and bending moments $M_{c,r}(t)$, $M_{a,r}(t)$, for which it is characteristic that by writing equilibrium and compatibility equations and the constitutive laws for the two materials, the problem is governed by a system of two simultaneous differential equations, which have been derived and solvedFig.1). Further development of rheology as a fundamental science and its application to concrete as well as a great number of investigations in the field of creep of concrete have led to new formulations of the time-dependent behavior of concrete. These new formulations giving the relationship between $\sigma_c(t)$ and $\varepsilon_c(t)$ are formulated by integral equations, which present the basis of the theory of linear viscoelastic bodies. The integral-type creep law, i.e., the superposition equation for uniaxial prescribed stress history $\sigma(t)$, is expressed by:

$$\varepsilon_c(t,t_0)=\varepsilon^{sh}(t)+\sigma(t_0)J(t,t_0)+\int_{t_0}^{t}\frac{d\sigma(\tau)}{d\tau}J(t,\tau)d\tau\ . \qquad (1)$$

Since the solutions of structural creep problems with realistic compliance function , such as in ACI209R-92, EC2 and G&L models for creep of concrete provisions cannot be performed analytically and require a deep knowledge in the higher level of mathematics theory(integral equations of Volterra, numerical solutions of such of type equations, using formulae of quadratures - such as: trapecoidal rules, Simpson, Chebyshov, Gaus, Euler-Gregory -Macloren, simplified approximate methods have been favoured by designers.

2.2. Formulation of the age-adjusted effective modulus method

However, in order to avoid the mathematical problems in solving of the integral equations of Volterra for treating the problem connected with the creep of concrete structures, it has been revised the integral relationship into new algebraic stress-strain relationship:

$$\varepsilon_{ct}=\frac{\sigma_{c0}}{E_{c0}}\left[1+\varphi_t\right]+\frac{\sigma_{ct}-\sigma_{c0}}{E_{c0}}\left[1+\rho\varphi_t\right]. \qquad (2)$$

where ρ is the relaxation coefficient known from Trost-Zerna works[17]. When more extensive test date and data of long duration became available and were systematically analysed from Bažant[4, 8], it turned out that the afore-mentioned theory leading to differential equations are overall not more accurate than the effective modulus methods(Partov and Kantchev-[10,11,12]), which leads to algebraic linear equations with respect to time *t*. According Bažant and Jirasek[8] none of them is sufficiently accurate compared to the computer solutions for e realistic (un -simplified) compliance function based on long – time –measurements with a broad range of ages at loading. A remedy that is sufficiently accurate in most basic situations we can found in the *age-adjusted effective modulus method*, proposed and mathematically proven by Bažant[8], as a modification and refinement of the relaxation method, semi-empirically developed by Trost[14,17]. The *age-adjusted effective modulus (AAEM) method* is formulated for a one-step loading history. The AAEM methods is development from Bažant[4,8] as follows. The history of stress and strain between the initial and the current state is approximated by a linear combination of creep at constant stress and relaxation at constant strain. Let's now recall that the strain α, suddenly applied at time t_1 and subsequently kept constant, produces stress history: $\sigma(t)=\alpha.R(t,t_1)$, where $R(t,t_1)$ is the relaxation function[8]. Also, the stress β, suddenly applied at time t_1 and subsequently kept constant, produces strain history: $\varepsilon(t)=\beta.J(t,t_1)$, where $J(t,t_1)$ is the compliance function. Now using the principle of superposition he state, that the strain and stress histories correspond to each other, what mean that the two expressions satisfy the viscoelastic constitutive equations: $\varepsilon(t)=\alpha+\beta.J(t,t_1)$; $\sigma(t)=\alpha.R(t,t_1)+\beta$; by: $t\geq t_1$. After that, he admit that the coefficient α and β can be expressed in terms of the stress values $\sigma(t_1^+)=\sigma_1$, just after load application, and $\sigma(t)=\sigma_1+\Delta\sigma$, at current time t. Substituting these values into $\sigma(t)=\alpha.R(t,t_1)$ and recalling that: $R(t_1,t_1)=1/J(t_1,t_1)=E(t_1)$ = elastic modulus at age t_1, we obtained a set of two linear equations for α and $\beta:\alpha.E(t_1)+\beta=\sigma_1$; $\alpha.R(t,t_1)+\beta=\sigma_1+\Delta\sigma$; from which it is possible to calculate the coefficients: $\alpha=\Delta\sigma/(R(t,t_1)-E(t_1)$ and $\beta=\left[\Delta\sigma.E(t_1)/E(t_1)-(R(t,t_1)\right]+\sigma_1$. Using this results, it is easy to evaluate from $\varepsilon(t)=\alpha+\beta.J(t,t_1)$ the initial (elastic) strain:

$$\varepsilon_1=\varepsilon(t_1^+)=\alpha+\beta.J(t_1,t_1)=\alpha+\beta/E(t_1)=\sigma_1/E(t_1)\,. \qquad (3)$$

and the strain increment:

$$\Delta\varepsilon = \varepsilon(t) - \varepsilon(t^{+}{}_1) = \beta\left[J(t,t_1) - J(t_1,t_1)\right] == \Delta\sigma\frac{E(t_1).J(t,t_1)-1}{E(t_1)-R(t,t_1)} + \sigma_1\left[J(t,t_1) - 1/E(t_1)\right] \cdot \tag{4}$$

If the expressions : $E(t_1).J(t,t_1)-1$ is recognized as the creep coefficient: $\varphi(t,t_1)$. So, if we introduce the so called *age-adjusted effective modulus as follows:*

$$E^{//}(t,t_1) = \frac{E(t_1)-R(t,t_1)}{E(t_1).J(t,t_1)-1} = \frac{E(t_1)-R(t,t_1)}{\varphi(t,t_1)}. \tag{5}$$

If we replace: $\sigma_1 / E(t_1)$ by ε_1 according the above mentioned formulae, the formulae (4) assumes a convenient form:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E^{//}(t,t_1)} + \varepsilon_1.\varphi(t,t_1). \tag{6}$$

This is the fundamental equation of AAEM, stating by brilliant way from Bažant[8], that the increment of strain over the interval (t_1,t) is equal to the increment of stress divided by the effective modulus plus the initial(elastic) strain multiplied by the creep coefficient.
For convenience the *age-adjusted effective modulus, whose primary definition is (5), can be expressed in the form:*

$$E^{//}(t,t_1) = \frac{E(t_1)}{1+\chi(t,t_1)\varphi(t,t_1)}; \tag{7}$$

which represent a correction of the effective modulus. This has the advantage that the so- called aging coefficient:

$$\chi(t,t_1) = \frac{E(t_1)}{E(t_1)-R(t,t_1)} - \frac{1}{\varphi(t,t_1)}. \tag{8}$$

varies relatively little (usually from 0,5 to 1,0, where the 0,8 is as the most typical value).
The values $\chi = 1$ characterizes the limiting case of non-aging material well. Indeed if there is no aging, as in the case of shorter creep durations for concrete loaded at old age, the optimal value of χ is closed to 1(about 0,992), and $E^{//}$ is nearly equal to $E_{eff.}$. Tables of χ computed for certain compliance functions, have been included in ACI Committee 209 design recommendations. To avoid a computer solution of the relaxation function $R(t,t_1)$ for a given compliance function $J(t,t_1)$, one may use the following semi-empirical approximate formula with correct asymptotic properties [8]:

$$R(t,t_1) = \frac{0,992}{J(t,t_1)} - \frac{0,115}{J(t,t-1)}\left[\frac{J(t-\Delta,t_1)}{J(t,t_1+\Delta)} - 1\right]; \quad \Delta = \frac{t-t_1}{2} \tag{9}$$

For normal concrete the error of this formula(recommended for calculating χ in CEB-FIP Model Code 1990, (Eqs.5.8-7 and 5.8-3) is generally under 1%)[8].

By using algebraic approach a new simpler forms for (1) are obtained from Bažant[8]. His methods are based on the hypothesis that the strain in the concrete fibers can be considered as a linear function of the creep coefficient. This permits transforming (1) in to (10):

$$\varepsilon_c(t,t_0)=\varepsilon^{sh}(t)+\sigma_c(t_0)\left[\frac{1}{E_c(t_0)}+\frac{\varphi_{28}(t,t_0)}{E_{c28}}\right]+\left[\sigma_c(t)-\sigma_c(t_0)\right]\left[\frac{1}{E_c(t_0)}+\frac{\chi(t,t_0)\varphi_{28}(t,t_0)}{E_{c28}}\right]. \quad (10)$$

where:

$$\chi(t,t_0)=\frac{E_c(t_0)}{E_c(t_0)-R(t,t_0)}-\frac{E_{c28}}{E_c(t_0)\varphi_{28}(t,t_0)}. \quad (11)$$

is the aging coefficient; $\varphi(t,t_0)$ - the creep coefficient ; $R(t,t_0)$ - relaxation function, i.e., the stress response to a constant unit strain applied at the time t_0; E_c - the elastic modulus of concrete at 28 days. The age-adjusted effective method (AAEM) directly assumed the expression provided by (11) for the aging coefficient. In this case, it is necessary to evaluate previously the relaxation function $R(t,t_0)$. This function is calculated numerically by applying the step-by-step procedure of the general method to the integral type relation between the creep and the relaxation function. The main advantage of the method, consequent to the adoption of the algebraic formulation instead of the hereditary integral formulation for the constitutive viscoelastic equations of the different parts, consists in avoiding the need to store the time history of each sub-element. One other advantage of the AAEM method is the fact that $\chi(t,t')$ varies relatively little with the age t' for sufficiently long elapsed times. Its long-term values are in a range between 0,5 and 1,0, the most common values, for typical values of t' and other influencing parameters, being contained in a narrower range between 0,7 and 0,9, in particular for B3 and GL2000 creep prediction models. It is often adequate to use the value $\chi = 0,8$. In the papers [10,11,12] using the mathematical model in Fig. 1, the authors introduce the system of linear Volterra integral equation of second kind and obtain the results from their numerical solutions. The kernels of the integral equations contain the respective creep functions corresponding to the model EC2 [1].

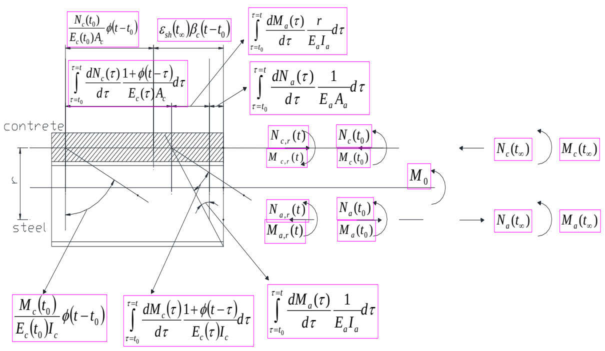


Figure 1. Mathematical model

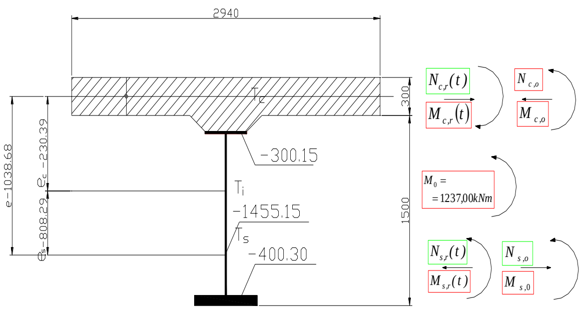


Figure 2. Composite beam

3. BASIC ASSUMTION AND MATERIAL CONSTITUTIVE RELATIONSHIP

The hypotheses in the elastic analysis of composite steel-concrete sections with stiff (rigid) shear connectors are assumed as following in [10,11,12,16].:

4. BASIC EQUATION EQUILIBRIUM

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time t = 0 with $N_{c,0}$, $M_{c,0}$, $N_{a,0}$, $M_{a,0}$ and with $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{a,r}(t)$, $M_{a,r}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete. For a composite bridge girder with $J_c = \dfrac{A_c(nI_c)n}{A_s I_s} \le 0.2$ according to the suggestion of (Sonntag 1951) we can write the equilibrium conditions in time t as follows:

$$N(t) = 0; \quad N_{c,r}(t) = N_{a,r}(t); \tag{12}$$

$$\sum M(t) = 0; \quad M_{c,r}(t) + N_{c,r}(t) r = M_{a,r}(t). \tag{13}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (12), (13) are not sufficient to solve it. It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t (Fig. 1).

5. DERIVING THE GENERALISED MECHANIC-MATHEMATICAL MODEL USING INTEGRAL EQUATIONS OF VOLTERRA ACCORDING EC2

Using the strain compatibility on the contact surfaces between the concrete and steel members and compatibility of curvatures when $\tau = t$, for constant elasticity module of concrete, after integrating the two equations by parts transforming the integrals into Riemann ones and using the (12) and (13) for assessment of normal forces $N_{c,r}(t)$ and bending moment $M_{c,r}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{c,r}(t) = \lambda_N \int_{t_0}^{t} N_{c,r}(\tau) \frac{d}{d\tau}\left[1 + \varphi\phi_{RH}\beta(f_{cm})\beta(\tau)\beta(t-\tau)\right] d\tau + \lambda_N N_{c,0}\phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0) + ; \tag{14}$$
$$+\lambda_N N_{sh}\beta_c(t-t_0)$$

$$M_{c,r}(t) = \lambda_M \int_{t_0}^{t} M_{c,r}(\tau) \frac{d}{d\tau}\left[1 + \phi_{RH}\beta(f_{cm})\beta(\tau)\beta_c(t-\tau)\right] d\tau + \lambda_M M_{c,0}\phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0) - ; \tag{15}$$
$$-\lambda_M \frac{E_c I_c}{E_a I_a} N_{c,r}(t) r$$

In which: $\lambda_N = \left[1 + \dfrac{E_c A_c}{E_a A_a}\left(1 + \dfrac{A_a r^2}{I_a}\right)\right]^{-1}$; $\lambda_M = \left[1 + \dfrac{E_c I_c}{E_a I_a}\right]^{-1}$., where: the functions:

$\lambda_N N_{c,0}\Phi(t_c)\beta_c(t-t_0)$, $\lambda_M M_{c,0}\Phi(t_c)\beta_c(t-t_0)$, $\dfrac{d}{d\tau}(1+\Phi(t_c)\beta(t-\tau)$ - are given.

6. DERIVING THE GENERALISED MECHANIC-MATHEMATICAL MODEL USING ALGEBRAIC EQUATIONS ACCORDING AAEM METOD OF BAŽANT

Using the above mentioned approach, for constant elasticity module of concrete for assessment of normal forces $N_c(t)$ and bending moment $M_c(t)$ two algebraic expressions are derived:

$$N_c(t) = \frac{N_c(t_0)\varphi(t,t_0)}{\left[\lambda_N^{-1} + \chi(t,\tau)\varphi(t,\tau)\right]} = \frac{N_c(t_0)\varphi(t,t_0)\lambda_N}{\left[1 + \chi(t,\tau)\varphi(t,\tau)\lambda_N\right]}; \tag{16}$$

$$M_c(t) = \frac{M_c(t_0)\varphi(t,t_0)\lambda_M}{\left[1+\chi(t,\tau)\varphi(t,\tau)\lambda_M\right]} - \frac{N_c(t) r \lambda_M}{\left[1+\chi(t,\tau)\varphi(t,\tau)\lambda_M\right]} \frac{E_c J_c}{E_a J_a}. \quad (17)$$

7. NUMERICAL EXMPLE

The two methods presented in the previous paragraph is now applied to a simply supported beam, subjected to a uniform load, whose cross section is shown in Fig. 2. The following parameters are chosen according EC2 model.

$$E_c = 3{,}2.10^4 \ MPa,\ E_a = 2{,}1.10^5 \ MPa,\ A_c = 8820\,cm^2,\ A_a = 383{,}25\,cm^2, n = \frac{E_a}{E_c} = 6{,}56$$

$$I_c = 661500\,cm^4,\ I_a = 1217963{,}7\,cm^4,\ r_c = 23{,}039\,cm,\ r_a = 80{,}829\,cm,\ r = 103{,}868\,cm,$$

$$A_i = 2453{,}05\,cm^2,\ I_i = 4536360{,}758\ cm^4 . M_0 = 1237\,kNm,\ N_{c,o} = 846{,}60\,kN,\ M_{c,o} = 27{,}56\,kNm$$

$$M_{a,o} = 330{,}13kNm,\ \lambda_N = \left[1+\frac{E_c A_c}{E_a A_a}\left(1+\frac{A_a r^2}{I_a}\right)\right]^{-1} = 0{,}060545358,\ \lambda_M = \left[1+\frac{E_c I_c}{E_a I_a}\right]^{-1} = 0{,}922950026$$

$$h_0 = 2AC/u = 300\ mm;\ \beta_H = 150\left[1+\left(1.2*80/100\right)^{18}\right]h_0/100+250 = 915{,}82 < 1500$$

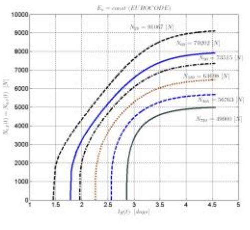
$$\beta(f_{cm}) = \left.\frac{5.3}{(f_{cm}/10)^{0.5}}\right|_{f_{cm}=30} = 3.06 \qquad \beta(t_0) = \left.\frac{1}{0.1+(t_0)^{0.2}}\right|_{t_0=60} = 0{,}4223;$$

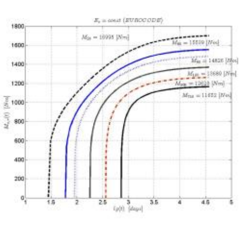
$$\phi_{RH} = 1+\left.\frac{1-RH/100}{0.46\sqrt[3]{(h_0/100)}}\right|_{RH=80,h_0=300} = 1{,}30146; \qquad \phi_0 = \phi_{RH}\beta(f_{cm})\beta(t_0) = \mathbf{1{,}6817;}$$

$$\beta_c(36500-60) = 0{,}9925811; \qquad \phi_{t=36500} = \phi_0\beta_c(36500-60) = 1{,}669242;$$

7.1. Numerical solution using integral equation of Volterra

In Figures 3,4 and 5 it is shown the values of normal forces and bending moments in time *t*. A numerical method for time-dependent analysis of composite steel-concrete sections according EC2, models is develop using MATLAB code and numerical algorithm which was successfully applied to a simple supported beam. These numerical procedures, suited to a PC, are employed to better understand the influence of the creep of the concrete in time-dependent behaviour of composite section. For a good accuracy of the time values, the numerical results are presented on logarithmic time scales. The choice of the length of time step of the proposed numerical algorithm is based on numerous numerical experiments with different steps (seven, three and one days). So we conclude that good results can be achieved from practical point of view with three day step. For our purpose we consider a period of about 100-102 years. For the service load analysis, the proposed numerical method makes it possible to follow with great precision the migration of the stresses from the concrete slab to the steel beam, which occurs gradually during the time as a result of the creep of the concrete.





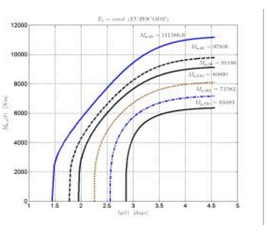


Figure 3. $N_{c,r}(t) = N_{a,r}(t)$ Figure 4. $M_{c,r}(t)$ Figure 5. $M_{a,r}(t)$

in time in time $t = 36528 - 37230$ days

7. 2. Prediction of Concrete creep effect Using AAEM

Let,s consider the next following initial data: $t_0 = 60$ days; $t_\propto = 36561$ days; $E_c(t_0) = 32000 MPa$; The creep coefficient:

7.2.1.

$$\phi(t_\propto = 36561, t_0 = 60) = \phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0) = 1,30146.3,06.\frac{1}{0,1+(t_0=60)^{0,2}}\left[\frac{t-t_0 = 36561-60 = 36501}{\beta_H+(t-t_0) = \beta_H + 36501 = 915,8215+36501}\right]^{0,3} = 1,6693766031$$

7.2.1.1. $J(t = 36561, t_0 = 60) = \dfrac{1+\varphi(t,t_0)}{E_{cmt_0}} = \dfrac{1+1,669376031}{32000} = 0,000083418$;

7.2.2. $\varphi(t, t_0 = (t-1)) = (36561, 36500) = 1,30146.3.06\dfrac{1}{0,1+(36561)^{0,2}}\left[\dfrac{t-t_0 = 36561-36560 = 1}{\beta_H+(t-t_0) = 915,82+1}\right]^{0,3} = 0,062170499;$

7.2.2.1.

$J(t = 36561, t-1 = 36560) = [1+\varphi(t,t-1)]/E_{cmt_0} = [1+0,062170419]/32000 = 0,000033192$;

7.2.3. $\Delta = t - t_1/2 = 36561 - 60/2 = 18250,5$;

7.2.3.1. $\varphi(t-\Delta, t_1) = \varphi(36561-18250,5) = \varphi(18310,5) = 1,30146.3,06.\dfrac{1}{0,1+60^{0,2}}\left[\dfrac{t-\Delta, t_0 = 18310,5-60 = 18250,50}{915,82+18250,5}\right]^{0,3} = 1,658949944;$

7.2.3.2. $J(t-\Delta, t_1) = [1+\varphi(t-\Delta,t_1)]/E_{cmt_0} = [1+1,658949944]/32000 = 0,000083092;$

7.2.4. $\phi(t, t_1+\Delta) = \varphi(t = 36561, t_1 = 60+18250,5 = 18310,5) =$

$$= 1,30146.3,06.\frac{1}{0,1+18310,5^{0,2}}\left[\frac{36561-18310,5 = 18250,5}{915,8215+18250,5 = 19166,3215}\right]^{0,3} = 0,543471329;$$

7.2.4.1. $J(t, t_1+\Delta) = [1+\varphi(t,t_1+\Delta)]/E_{t_0} = [1+0,543471329]/32000 = 0,000048233$;

7.2.4.2. $R(t,t_0)$**- relaxation function:**

$$R(t,t_0) = \frac{0,992}{J(t,t_0)} - \frac{0,115}{J(t,t-1)}\left[\frac{J(t-\Delta,t_1)}{J(t,t_1+\Delta)} - 1\right] = 9387,913527$$

7.2.5. The aging coefficient:

$$\chi(t,t_0) = \chi(36561,60) = \frac{E(t_0)}{E(t_0)-R(t,t_0)} - \frac{1}{\phi(t,t_0)} = \frac{32000}{32000-9387,913527} - \frac{1}{1,669376029} = 0,816146157$$

Then: $N_c(t) = \dfrac{N_c(t,t_0).\varphi(t,t_0).\lambda_N}{1+\chi(t,t_0).\varphi(t,t_0)\lambda_N} = \dfrac{84660.1,669376029.0,060545358}{1+0,816146157.1,669376029.0,060545358} = 7904,770564 daN$ (according AAEM).

7.2.6. Calculating $M_c(t)$ using the formulae:

$$M_c(t) = M_c(t_0).\varphi(t,t_0).\lambda_M / 1+\chi(t,t_0).\varphi(t,t_0)\lambda_M - N_c(t).r.\lambda_M\frac{E_cI_c}{E_aI_a}/1+\chi(t,t_0).\varphi(t,t_0).\lambda_M = 1603,179037 daNm.$$

7.2.7. Calculating $M_a(t)$ using the formulae:

$$M_a(t) = M_c(t) + N_c(t).r = 1603,179037 + 7904,770564.1,0387 = 9814,071969 daNm$$

Comparisons between the results obtained from the numerical solution and AAEM methods are as follows $N_c(t)$ =7904,770 daN (by AAEM) and $N_c(t)$ =7920,2 daN (by numerical method); (Δ =0,1948%). $M_c(t)$ =1603,179 daNm (by AAEM) and $M_c(t)$ = 1553,90 daNm (by numerical method) (Δ =3,073%). $M_a(t)$ = 9814,0719 daNm (by AAEM) and $M_a(t)$ = 9780,60 daNm (by numerical method). (Δ =0,341%).

8. COMPARISON WITH EFFECTIVE MODULUS METHODS (EMM)

This method uses the Dischinger's idea for applying in the calculation the ideal (fictitious) modulus of elasticity [6,7]: $E_{ci} = \frac{E_{cm}}{1+\psi_L \phi_t} = \frac{E_{cm}}{1+1{,}1\phi_t}$; where ϕ_t is a final creep coefficient of concrete.

It is applied here by the authors of the paper to solve practical case shown in Fig. 2. The results obtained by: (EMM) of Dischinger, numerical solution and analytical AAEM method of Bažant are illustrated in tables 1 and 2 and 3.

Table 1. Dimensions of steel and composite beams

Type of beams		steel	Composite (in $t_0 = 0$)	Composite (in $t = \infty$)	
height	h_i	1500	1800	1800	mm
area	A_i	38325	172725	85689	mm^2
Static moment to down surface	S_{y0}	23428688	245188688	101578534	mm^3
Gravity center	e_{top}	888,7	380,5	614,6	mm
Gravity center	e_{bottom}	611,3	1419,5	1185,4	mm
Moment of inertia	$I_{i,y}$	12179635497	45260127815	35415902690	mm^4
Section modulus	$W_{i,y,ct}$		-118959133	-57690019	mm^3
Section modulus	$W_{i,y,cb}$		-562462122	-112824475	mm^3
Section modulus	$W_{i,y,at}$	13592026	-562462122	-112824475	mm^3
Section modulus	$W_{i,y,ab}$	19759036	31883835	29866674	mm^3

Table 2. The stresses in upper and lower fibres of concrete plate and steel beam in time t_0 and t_∞ according EMM of Dischinger

Stress in time t_0	t_0 = 60 days	Stress in time t_∞ EMM(Dischinger)	t_∞ = 36561 days
M_0 (kNm)	1237	M_0 (kNm)	1237
$n_0 = E_a / E_{cm}$	6,56	$n_L = n_0(1+\psi_L \phi_t)$; $\psi_L = 1{,}1$	18,5748
$\sigma_c^{top} = M / W_{i,y,ct} / n_0$ (MPa)	-1,58	$\sigma_c^{top} = M / W_{i,y,ct} / n_L$ (MPa)	-1,15436
$\sigma_c^{bottom} = M / W_{i,y,cb} / n_0$ (MPa)	-0,33	$\sigma_c^{bottom} = M / W_{i,y,cb} / n_L$ (MPa)	-0,590
$\sigma_a^{top} = M / W_{i,y,at}$ (MPa)	-2,19	$\sigma_a^{top} = M / W_{i,y,at}$ (MPa)	-10,96
$\sigma_a^{bottom} = M / W_{i,y,ab}$ (MPa)	38,708	$\sigma_a^{bottom} = M / W_{i,y,ab}$ (MPa)	41,41

Table 3. The stresses in upper and lower fibers of concrete plate and steel beam in time t_∞ according Numerical Methods and AAEMM(Bažant)

Stress in time t_∞ according: (NUMERICAL METHOD)	$t_0 =$ 60 days $t_\infty =$36561days	Stress in time t_∞ according: AAEMM(Bažant)	t_∞ = 36561 days
M_0 (kNm)	1237	M_0 (kNm)	1237
$\sigma_c^{up} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-1,139	$\sigma_c^{up} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-1,128
$\sigma_c^{down} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-0,596	$\sigma_c^{down} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-0,608
$\sigma_a^{up} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	-11,394	$\sigma_a^{up} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	-11,41
$\sigma_a^{down} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	41,615	$\sigma_a^{down} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	41,387

The stresses in upper and lower fibres of concrete plate and steel beam we obtain in time t_∞ using the formulae which is known from the engineering discipline (strength of material) as follows: $\sigma_c^{up/down} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ and $\sigma_a^{up/down} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$, where: $N_c(t_\infty) = N_c(t_0) - N_{c,r}(t_\infty)$; $N_a(t_\infty) = N_a(t_0) + N_{a,r}(t_\infty)$; $M_c(t_\infty) = M_c(t_0) - M_{c,r}(t_\infty)$; $M_a(t_\infty) = M_a(t_0) + M_{a,r}(t_\infty)$

z_c and z_a – distance from the mass center of concrete plate and steel beam to their upper and lower fibres.

9. CONCLUSION

The results obtained by the AAEM method of Bažant are completely comparable with the results based on numerical method according to the EC2 provision and the method of Dischinger (EMM) recommend in EC4.

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